MATHEMATICS
Through Narrative

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*D*eadline.
Zork.
Enchanter.
The Hitchhiker's
Guide to the Galaxy.
Trinity. These names evoke the golden age of one of the first truly popular PC game genres: interactive fiction (IF). In the 1980s, when computer graphics were still quite primitive, many of the best (and best-selling) computer games were interactive stories. The player played the game by typing what he or she wished to do. For example, here is the classic beginning to *Zork I*, one of the best-selling computer games of the early 1980s.

```
WEST OF HOUSE
You are standing in an open field west
of a white house, with a boarded front
door.
A small mailbox is here.

> examine mailbox
The small mailbox is closed.

> open mailbox
Inside the mailbox is: a leaflet

> read leaflet
"WELCOME TO ZORK!
ZORK is a game of adventure, danger,
and low cunning. In it you will explore
some of the most amazing territory ever
seen by mortals. No computer should be
without one!"
```

Today it is difficult to communicate the sense of awe and wonder these lines evoked in the 1980s. A story that you could create by your choice of actions? Virtually unheard of.

In addition to the story, much of the fun of games like *Zork* also lay in solving puzzles: figuring out a riddle, getting past a locked door, or even defeating a thief who keeps stealing your valuables. As interactive fiction evolved as a genre, these puzzles often became more and more sophisticated. *Zork Zero*, for example, includes the towers of Hanoi recursion problem. *Spellbreaker* features the famous 12-coin logic puzzle, in which you try to determine the one coin that has a different weight than the others using only three weighings on a set of scales.

Trinity, though, features a clever puzzle that's more than just an implementation of a classic problem in logic. At some point in the game you find yourself in a garden.

```
ARBORETUM
A spectacular pergola of arborvitaes
arches over your head like a great
green Ferris wheel. Its tangled
surfaces are peculiarly twisted, making
it difficult to tell where the inside
ends and the outside begins.
Steep, leafy tunnels curve up into the
pergola to the north and south. Other
paths lead east and west, into the
surrounding hedge.
An abstract sparkling sculpture stands
between the tunnels. The words FELIX
KLEIN 1849-1925 are inscribed on the
base.

> examine sculpture
The sparkling sculpture looks like a
crystalline bottle, eight feet high,
with a polished surface that twists in
impossible curves. It's hard to tell
where the outside of the sculpture ends
and the inside begins.
The words FELIX KLEIN 1849-1925 are
inscribed on the base.
```

It turns out that the sculpture is a Klein bottle. A Klein bottle is a topological object that is, like a Möbius band, one-sided, but unlike a Möbius band, has no edge (see figure 3). Without leaving the surface of the Klein bottle, an ant could walk from a point on the "outside" to the same point on the "inside." Also, if the ant were to carry an *xy*-coordinate frame with him on this journey, it would have the opposite orientation when it arrived at its starting location.

As the text hints, the arbor itself is a Klein bottle as well. You can realize this by walking all the way through the arbor's tunnels from one end to the other. When you return to the arboretum after doing so, the game's text about the sculpture now reads:

```
An abstract sparkling sculpture stands
between the tunnels. The words 5291-9481
NIELK XILEF are inscribed on the base.
```

This is the player's indication as to the properties of a Klein bottle: traversing it reverses the orientation of everything in the game relative to you. This is more than just a mathematical curiosity, however: The Klein arbor is necessary to solve one of the puzzles in the game. One of the objects you would be carrying at this point has left-handed threads, but elsewhere in the game you need to screw it into a hole that has right-handed threads. The solution to the puzzle is to walk all the way through the arbor while carrying the object, thus reversing the orientation of everything in the game relative to you and the object. You can then screw the object into the hole (because the threads now match!), walk back through the arbor to restore everything to its proper orientation relative to you, and continue through the rest of the game.

I first played *Trinity* at age 13, and I was fascinated with the Klein bottle puzzle. It was probably my first experience with an advanced mathematical concept; certainly, it was even more interesting than what I was learning in school at the time. Also, after I figured out the puzzle I was left with a sense of accomplishment, a sense of wonder at such an interesting object, and—because I had to think through how to use an advanced mathematical concept on my own—a permanent memory of one property of Klein bottles.

Trinity's Klein arbor wasn't the only piece of advanced mathematics I saw in some of those old interactive fiction games. *Beyond Zork* (also written by *Trinity* author Brian Moriarty) features the following interaction.

```
SOUTH CHASM
You're shivering on the south edge of
a broad chasm. Clammy mists chill the
air, and the ground trembles with the
roar of a cataract.
Your heart sinks as you inspect the
crude rope bridge spanning the chasm. A
notice hangs near the bridge's entrance.

> read notice
The notice says,
ZENO'S BRIDGE
Cross at thy Own Risk

> go north
HALFWAY TO THE NORTH END

> go north
3/4 OF THE WAY TO THE NORTH END

> go north
7/8 OF THE WAY TO THE NORTH END
```

At this point—even if they have never heard of Zeno's paradox—the player starts to realize what's going on and tries to return to the start of

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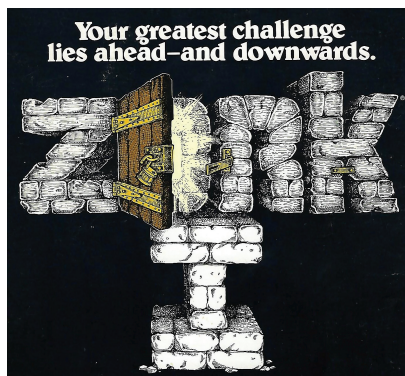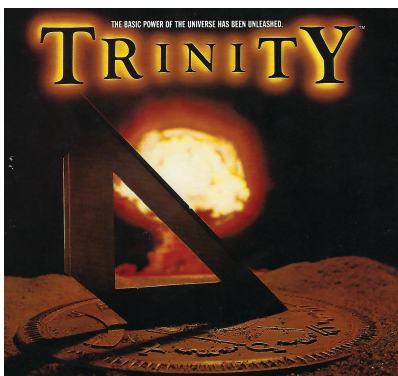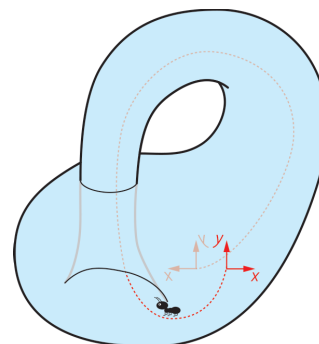**Figure 1.** Zork I.

Figure 2. Trinity.

Figure 3. A Klein bottle is a one-sided surface.

the bridge. But the game takes Zeno's paradox seriously:

```
> go south
9/16 OF THE WAY TO THE SOUTH END

> go south
25/32 OF THE WAY TO THE SOUTH END
```

The game does allow the player to leave the bridge, but simply going north or south won't work.

My first time playing *Beyond Zork* (in 1987) was the first time I had heard of Zeno's paradox.

By around 1990, computer game demand had shifted decisively away from text-only games because of improvements in graphics technology. Since then, interactive fiction has for the most part not been commercially viable. However, there has continued to be a dedicated hobbyist community supporting the creation and distribution of interactive fiction for free. Many of the members of this community are themselves scientists or mathematicians, and scattered through the many hobbyist IF games produced over the past 30 years are references to mathematical and scientific concepts.

Erehwon (1999) includes puzzles involving a Klein desert and a Hamiltonian circuit, not to mention Platonic solids and a Ramanujan joke. *Curses!* (1993) features a "Universe Maintenance Room" with a dial set to "1.055 or thereabouts" (Planck's constant). Setting the dial to any other number causes the universe to become incorporeal (and this, unsurprisingly, also ends the game). *Risorgimento Represso* (2003) requires the player to create gunpowder by mixing the

Figure 4. Beyond Zork.

proper elements, which the player learns from its chemical recipe (provided in-game).

A New Interactive Fiction Mathematical Adventure

Inspired by the Klein arbor puzzle in *Trinity*, I recently wrote my own piece of IF, *A Beauty Cold and Austere* (*ABCA*). The puzzles in *ABCA* are almost completely mathematical. My intent with the game, much like that Klein arbor and Zeno's bridge in *Beyond Zork*, is to allow people to explore mathematical concepts through narrative. My hope is that presenting the ideas in an interactive, game-based format will make the concepts more accessible and less intimidating. The game is educational, in the broad sense: It cannot substitute for a course in mathematics, but hopefully people can learn some mathematics while having fun tackling the game's challenges.

For example, here is an early puzzle in the game.

```
LIBRARIAN'S OFFICE
This appears to be the head librarian's
office in the Great Library of
Alexandria. Scrolls litter the floor. The
main part of the library is north of you.
The numbers from 1 to 100 are carved on
the wall here, in some kind of puzzle.

> examine puzzle
On the wall are carved numbers from
1 to 100, in ten rows of ten each.
It looks like you could push any of
the numbers. Next to the numbers is
a switch, with two settings: "Remove
Number" and "Remove Larger Multiples of
the Number." The switch is currently
set to "Remove Number," although you
could easily move it to the other
setting by flipping the switch. The
numbers currently look like this:
  1  2  3  4  5  6  7  8  9  10
 11 12 13 14 15 16 17 18 19 20
 21 22 23 24 25 26 27 28 29 30
 31 32 33 34 35 36 37 38 39 40
 41 42 43 44 45 46 47 48 49 50
 51 52 53 54 55 56 57 58 59 60
 61 62 63 64 65 66 67 68 69 70
 71 72 73 74 75 76 77 78 79 80
 81 82 83 84 85 86 87 88 89 90
 91 92 93 94 95 96 97 98 99 100
At the bottom is a challenge from the
librarian: "To access the map room,
leave just the primes between 1 and 100
by pushing only five numbers."
```
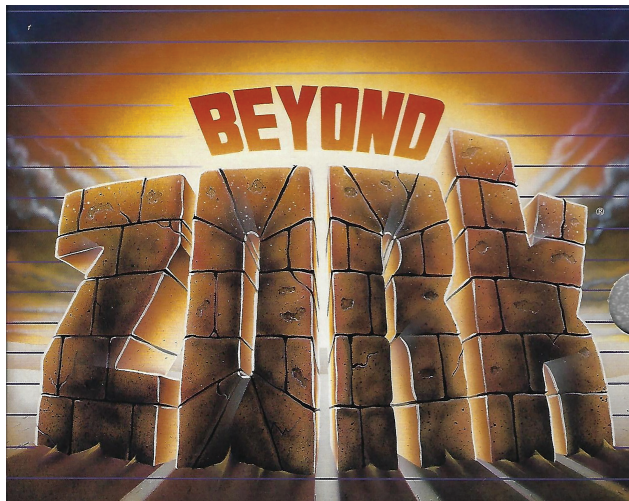
Players with more mathematical background will recognize that the puzzle requires the use of the sieve of Eratosthenes. However, many amateur players may not have seen the sieve before. Whether or not you have encountered the sieve, can you solve this puzzle? Can you generalize it to more rows?

Other puzzles in *A Beauty Cold and Austere* involve a square root that you can carry around and manipulate, a roller coaster track that can reform as the graph of an equation, and a transformation matrix that can change the shapes of objects. You'll also get to play poker with Pascal, beat an algebra challenge from al-Khwarizmi, and rescue Descartes from an *unthinkable* fate. You might even run across a math joke or two.

Tinkering is a great way to learn. When we play around with concepts, we come to understand them more deeply. Ideally, this is partly what homework is supposed to do, although we all know that in practice things don't generally work like that. However, mathematics presented as part of a challenge in a game becomes, for most of us, motivation enough to engage to a depth we never would with an assignment. This idea is not particularly novel or controversial; for example, Keith Devlin wrote an entire book on the subject: *Mathematics Education for a New Era: Video Games as a Medium for Learning* (CRC Press, 2011).

If the puzzles or story here intrigue you, I encourage you to play *A Beauty Cold and Austere*. Don't worry whether you have the right math background; I'm sure you do. Several students of mine served as beta testers or played the game after release. Once I even had to cut off a student discussion of the game in order to start class! At least one of my students played all the way through to the end and won it; another told me she was stuck on some puzzles but wanted to keep at it without using the hints or walkthrough. In addition, I entered *ABCA* in the 2017 Interactive Fiction Competition, which ran from October 1 to November 15, 2017. Despite being a math-heavy game in a general competition, it managed to place seventh out of 79 games.

A Beauty Cold and Austere is available for free through the Interactive Fiction Database. You can either play it online or (for faster play) download it and play it with an interactive fiction interpreter like Gargoyle or Lectrote.

Feel free to write me about the game; I'd love to hear reactions to it (including bug reports).

Good luck and have fun! ●

Figure 5. *A Beauty Cold and Austere.*

Michael Z. Spivey is a math professor at the University of Puget Sound. Writing A Beauty Cold and Austere *was one of the most intellectually changing and rewarding things he has ever done.*

Solution to "The Three Spheres"
(see page 13)

Notice that

$$\mathrm{Vol}\big(B - (M \cup S)\big) = \mathrm{Vol}\big((M - B) \cup (S - B)\big)$$

if and only if

$$\mathrm{Vol}(B) = \mathrm{Vol}(M) + \mathrm{Vol}(S),$$

and this occurs if and only if $r_B^3 = r_M^3 + r_S^3$ where r_B, r_M, and r_S, are the radii of the three spheres. But because the radii are integers, this equality is impossible by Fermat's last theorem!

The fact that the two values are off by 1, $150^3 = 3{,}375{,}000$ and $144^3 + 73^3 = 3{,}375{,}001$, (through a known "near miss" to Fermat's last theorem for $n = 3$) and the specific placements of the spheres are red herrings! (A hint might have been that "Tamref Ed Erreip" is "Pierre de Fermat" spelled backwards.)